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Methods for optimizing the parameters of a linear programming problem (LPP)

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Abstract **—** this article presents a solution to a linear programming problem and a method that is important for optimizing parameter definition. In this case, the term programming should be understood as planning, that is, it is necessary to find the optimal production plan in which revenues will be maximum or costs will be minimal. One of these methods is the simplex method, the so-called method of gradual improvement of the plan. The idea of the method is that we choose some initial reference plan. Then, based on checking certain conditions, we move on to the next better plan. This process continues until we reach the optimal plan. At the same time, at every step we find one of the reference plans.

Keywords **—** *linear programming problems, simplex method, optimal solutions.*

I. INTRODUCTION

In this article, we will consider the issues of studying the optimality of the formulation itself LPP. If there are disadvantages, change the task parameters in order to improve the production process and increase the income of the enterprise. We will limit ourselves here to considering these issues using specific examples, but the conclusions we draw will be general in nature and can be applied to a wide range of tasks [1], [2], [4].

II. MAIN RESULTS

Let us be given a linear programming problem of the following form

$$\begin{cases} 7x_1 + 2x_2 + 5x_3 \le 64 \\ 4x_1 + 9x_2 + 3x_3 \le 46 \\ 3x_1 + 4x_2 + 10x_3 \le 73 \end{cases}$$

$$x_j \ge 0,\ j = 1, 2, 3,$$

$$L = 15x_1 + 25x_2 + 10x_3 \to \max.$$

Let's add an artificial basis and create the first simplex table

1st table

C_i basis	basis	15	25	20	0	0	0	b_i	δ_i
		A_1	A_2	A_3	A_4	A_5	A_6		
0	A_4	7	2	5	1	0	0	64	32
0	A_5	4	9	3	0	1	0	46	5.1
0	A_6	3	4	10	0	0	1	73	18
	Δ_j	-15	-25	-20	0	0	0	0	

The solution corresponding to this table will be $x_1=0;\ x_2=0;\ x_3=0;\ x_4=64;\ x_5=46;\ x_6=73.$

Obviously, it is not optimal, as there is $\Delta_j < 0.$

Let's define the permissive column and the permissive element from the first table and proceed to filling in the second simplex of the table. It is performed according to the above method and we get the following table

2nd table

C_i basis	basis	15	25	20	0	0	0	b_i	δ_i
		A_1	A_2	A_3	A_4	A_5	A_6		
0	A_4	6.1112	0	4.3334	1	-0.2222	0	53.7778	12.41
25	A_2	0.444	1	0.3333	0	0.1111	0	5.1111	15.33
0	A_6	1.2224	0	8.6668	0	-0.4444	1	52.5556	6.064
	Δ_j	-3.89	0	-11.6675	0	2.7775	0	127.7775	

The solution corresponding to this table will be $x_4=53.7778;\ x_2=5.1111;\ x_6=52.5556;\ x_1=x_3=x_5=0$.

The profit with this plan will be $L=127.7775$. Since among Δ_j There are negative aspects, this plan is not optimal. We find the permissive column and the permissive row (they are marked with an arrow in the table) and the permissive element. After that, we proceed to the compilation of the following table, which will turn out as follows

3rd table

C_i basis	basis	15	25	20	0	0	0	b_i	δ_i
		A_1	A_2	A_3	A_4	A_5	A_6		
0	A_4	5.5002	0	0	1	0.0913	-0.5001	27.5001	4.9998
25	A_2	0.397	1	0	0	0.1282	-0.0385	3.0898	7.7829
20	A_3	0.141	0	1	0	-0.0513	0.1154	6.064	43.04
	Δ_j	-2.255	0	0	0	2.179	1.3455	198.525	

The solution corresponding to this table will be $x_4=27.5001;\ x_2=3.0898;\ x_3=6.064;\ x_1=x_5=x_6=0$. The profit under this plan will be $L=198.525$. This plan is not optimal yet, as there is $\Delta_1 < 0$. Let's make the following table according to the same rule as the previous ones and get the following table

C_i basis	basis	15	25	20	0	0	0	b_i
		A_1	A_2	A_3	A_4	A_5	A_6 ←	
15	A_1	1	0	0	0.1818	0.0035	-0.0909	4.9998
25	A_2	0	1	0	-0.0722	0.1268	-0.0024	1.1049
20	A_3	0	0	1	-0.0256	-0.0518	0.1282	5.359
	Δ_j	0	0	0	0.41	2.1865	1.1405	209.8

The solution corresponding to this table would be $x_1=4.9998;\ x_2=1.1049;\ x_3=5.359;\ x_4=x_5=$

$x_6=0$. The profit under this plan will be $L=209.8$. Since there are no negative numbers in this table, Δ_j this plan will be optimal. As noted earlier, the simplex table also provides a solution to the dual problem. They are obtained by line Δ_j under artificial bases. Thus, we can state $y_1=0.41;\ y_2=2.1865;\ y_3=1.1405$. One of the criteria for evaluating the correctness of the solution obtained is the duality theorem

$$L_{\max} = 15 \cdot 4.9998 + 25 \cdot 1.1049 + 20 \cdot 5.359 = 209.7995,$$

$$Q_{\min} = 64 \cdot 0.41 + 46 \cdot 2.1864 + 73 \cdot 1.1405 = 210.0755.$$

As you can see, these values are almost equal. A minor difference is due to the rounding in the calculations. Thus the problem is solved. Moreover, in this task there is a case As you can see, these values are almost equal. A minor difference is due to the rounding in the calculations. Thus the problem is solved. Moreover, in this task there is a case $n=m=3$, full utilization of all resources will be observed. It's like there aren't any problems left. Let's put the question another way. Where should we start if we want to develop production? To do this, we will first analyze the resulting solution. Let's try to identify a scarce resource, increasing the reserves of which can increase production. This is all still within the framework of a mathematical model. Intuitively, the conclusion suggests itself that the raw materials with the highest price will be in short supply. In our case, this is the second raw material. Let's try to increase the reserves of this resource. At the same time, we must comply with all the constraints of the task. Let's imagine the reserves of the second resource as $46+z$. Here z an unknown addition to the second resource. In this case, the task will take the form

$$\begin{cases} 7x_1 + 2x_2 + 5x_3 \le 64 \\ 4x_1 + 9x_2 + 3x_3 \le 46 + z \\ 3x_1 + 4x_2 + 10x_3 \le 73 \end{cases}$$

$$x_j \ge 0, j = 1, 2, 3,$$

$$L = 15x_1 + 25x_2 + 20x_3 \to \max.$$

Let's try to find a plan with the full use of all three resources, that is, all restrictions take the form of equality. Then, to determine the solution, we can use the method of solving systems of linear algebraic equations [3].

Let's define the inverse matrix of the system of these conditions. It will turn out in the form of

$$A^{-1} = \frac{1}{419}\begin{pmatrix} 78 & 0 & -39 \\ -31 & 55 & -1 \\ -11 & -22 & 55 \end{pmatrix}$$

Let's solve the system of conditions using the matrix method

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \frac{1}{419}\begin{pmatrix} 78 & 0 & -39 \\ -31 & 55 & -1 \\ -11 & -22 & 55 \end{pmatrix}\begin{pmatrix} 64 \\ 46+z \\ 73 \end{pmatrix} = \frac{1}{419}\begin{pmatrix} 4992 - 2847 \\ -1984 + 2530 + 55z - 73 \\ -704 - 1012 + 4015 - 22z \end{pmatrix} = \begin{pmatrix} 5.1193 \\ 1.1289 + 0.1313z \\ 5.4869 - 0.0525z \end{pmatrix}$$

Let's substitute these expressions into the objective function

$$L = 15 \cdot 5.1193 + 25 \cdot (1.1289 + 0.1313z) + 20 \cdot (5.4869 - 0.0525z) = \\ = 214.75 + 2.2325z.$$

From the conditions of non-negativity x_j should

$$\begin{cases} 1.1289+0.1313z \ge 0 \\ 5.4869-0.0525z \ge 0. \end{cases}$$

Which means that $z \le 104.5$. According to the expression L it can be seen, that it increases with increasing z. Therefore, the best option for z there will be a difference $z=104.5$. Thus, if the reserves of the second resource are increased to $b_2=150.5$ then the company's profit will increase to then the company's profit will increase to 448.046. This is more than twice as much as in the initial, initial case. In this case, the enterprise plan will be $x_1=5.1193;\ x_2=14.85;\ \ x_3=0$.

We can also define a solution to the dual problem in a new version. To do this, we will use the matrix method again. Since the matrix of conditions of the dual problem is the transposed

matrix of the direct problem, its inverse matrix will be the transposed matrix of the inverse matrix of the direct problem. Based on this matrix, we can find a solution to the dual problem. In this case, we get

$$\begin{pmatrix} y_1 \\ y_2 \\ y_4 \end{pmatrix} = \frac{1}{419}\begin{pmatrix} 78 & -31 & -11 \\ 0 & 55 & -22 \\ -39 & -1 & 55 \end{pmatrix}\begin{pmatrix} 15 \\ 25 \\ 20 \end{pmatrix} = \begin{pmatrix} 0.418 \\ 2.23 \\ 1.17 \end{pmatrix}$$

Substituting the resulting solution $y_1=0.418;\ y_2=2.23;\ y_3=1.17$ in the objective function of the dual problem, we obtain

$$Q_{\min} = 64 \cdot 0.418 + 150.5 \cdot 2.23 + 73 \cdot 1.17 = 447.762.$$

As we can see, the statement of the duality theorem is confirmed. The results of a detailed analysis based on one example allow us to draw conclusions and recommendations for a wide range of tasks. The main thing is that the DLP can also be optimized according to the conditions of the setting, that is, according to the parameters. In conclusion, let's look at another example, during the solution of which it will be possible to make a number of more useful recommendations necessary for applied tasks.

Consider the following LPP

$$\begin{cases} 5x_1 + 8x_4 + 3x_3 \le 45 \\ 10x_1 + 3x_2 + 4x_4 \le 98 \\ 2x_1 + 4x_2 + 9x_3 \le 107 \end{cases}$$

$$x_j \ge 0, j = 1, 2, 3$$

$$L = 20x_1 + 32x_2 + 8x_3 \to \max.$$

We will find the solution to the problem using the simplex method

1st table

C_i basis	basis	20	32	8	0	0	0	b_i	δ_i
		A_1	A_2	A_3	A_4	A_5	A_6		
0	A_4	5	8	2	1	0	0	45	5.625
0	A_5	10	3	4	0	1	0	98	32.67
0	A_6	2	4	9	0	0	1	107	26.75
	Δ_j	-20	-32	-8	0	0	0	0	

2nd table

C_i basis	basis	20	32	8	0	0	0	b_i	δ_i
		A_1	A_2	A_3	A_4	A_5	A_6		
32	A_2	0.625	1	0.25	0.125	0	0	5.625	
0	A_5	8.125	0	3.25	-0.375	1	0	81.125	
0	A_6	-0.15	0	8	-0.5	0	1	84.25	
	Δ_j	0	0	0	4	0	0	180	

In table 2, among Δ_j there are no negative ones. Therefore, the solution corresponding to this table is $x_2=5.625;\ x_5=81.125;\ x_6=84.25;\ x_1=x_3=x_4=0$ it will be optimal. The company's income will be $L = 32 \cdot 5.625 = 180$ a monetary unit. From the solutions of the dual problem, which are obtained in the second table $y_1=4;\ y_2=0;\ y_3=0$, it follows that the first resource is scarce, and the second and third resources are in abundance. This also follows from substituting the solution into the objective function of the dual problem

$$Q = 45y_1 + 98y_2 + 107y_3 = 180$$

and the limitations of the main task

$$\begin{cases} 5 \cdot 0 + 8 \cdot 5.625 + 2 \cdot 0 = 45 \\ 10 \cdot 0 + 3 \cdot 5.625 + 4 \cdot 0 = 16.875 < 98 \\ 2 \cdot 0 + 4 \cdot 5.625 + 9 \cdot 0 = 22.5 < 107. \end{cases}$$

Let's consider a method for increasing a scarce resource. Let's apply the methodology outlined above. Let's find the inverse matrix for the matrix of conditions

$$A^{-1} = \frac{1}{-533}\begin{pmatrix} 11 & -64 & 26 \\ -82 & 42 & 0 \\ 34 & -4 & -65 \end{pmatrix}.$$

Let's find the formula for solutions when increasing the supply of the first resource to 45+z

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \frac{1}{-533}\begin{pmatrix} 11 & -64 & 26 \\ -82 & 41 & 0 \\ 34 & -4 & -65 \end{pmatrix}\begin{pmatrix} 45+z \\ 98 \\ 107 \end{pmatrix} = \begin{pmatrix} 5.619 - 0.0206z \\ -0.615 + 0.1538z \\ 10.914 - 0.0638z \end{pmatrix}$$

from non-negativity x_j, it follows from these formulas

$$\begin{cases} z \le 272.76 \\ z \ge 3.9987 \\ z \le 171. \end{cases}$$

The general solution, the set of solutions, of this system of inequalities will be the interval [3.99; 171]. Substituting expressions x_j in the formula of the objective function, we obtain

$$L = 180.012 + 3.9992z \to \max .$$

III. CONCLUSION

Obviously, the optimal value is z will be $z=171$.

In this case, we get $L_{\max} = 863.8752.$ If we compare this value with If we compare this value with $L = 180$ in the initial formulation, we see the effectiveness of this approach. With an increase in the reserves of the first resource by 171 units, the surpluses of the second and third resources are fully connected to production, and the company's income increases almost fivefold.

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